LECTURE: 5-1 AREAS AND DISTANCES

Areas - The Big Question: How Might you Find Area Under a Curvy Curve?

Example 1: Divide the interval $[0, 1]$ into $n = 4$ sub-intervals of equal width. Then, use four rectangles to estimate the area under $y = x^2$ from 0 to 1.

(a) Using left endpoints.

(b) Using right endpoints.

To find the actual area we need to take the number of sub-intervals to ______________. To do this we need a general expression for the left or right estimate for any n. This process is rather tedious and we will soon learn how we can use Calculus to find area under curves without having to use this long, tedious process.

Example 2: Prove that the area under $y = x^2$ from 0 to 1 is $\frac{1}{3}$.

Upper and Lower Sums: In general, we form **lower** (and **upper**) **sums** by choosing the sample points x_i^* so that $f(x_i^*)$ is the minimum (and maximum) value of f on the ith sub-interval.

Example 3: Estimate the area under $f(x) = 2 + x^2$, $[-2, 2]$ with $n = 4$ using

(a) Upper Sums

(b) Lower Sums

Question: What type of behavior will guarantee that the left sum is an under-estimate and the right sum is an over-estimate?

Example 4: Find an expression for the area under the graph of $f(x) = \sqrt{x}$, $1 \leq x \leq 16$ as a limit. Do NOT evaluate the limit.

Example 5: Determine a region whose area is equal to the given limit.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5+\frac{2i}{n}\right)^{10}$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\sin\left(2+\frac{5i}{n}\right)$

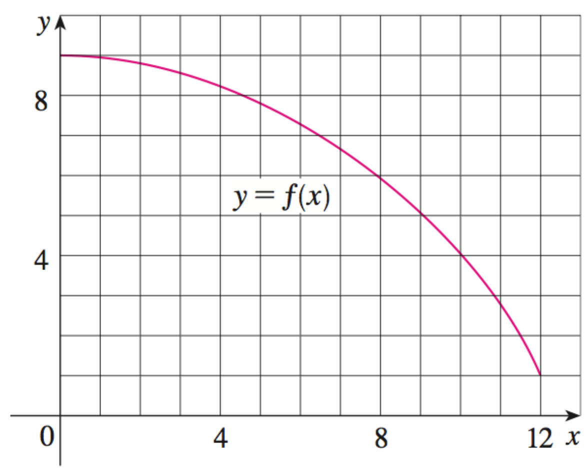
Example 6:

(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.

(i) L_6

(ii) R_6

(iii) M_6



(b) Is L_6 an underestimate or overestimate of the true area? Is R_6 an underestimate or overestimate of the true area?

(c) Which of the numbers L_6, R_6 or M_6 gives the best estimate? Explain.

Distances

Example 7: Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at 2 hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

t (h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

Example 8: Suppose the odometer on our car is broken and we want to estimate the distance driven over a 30 second time interval. We take speedometer readings every five seconds and then record them in the table below. Estimate the distance traveled by the car using a left sum and a right sum.

Time (s)	0	5	10	15	20	25	30
Velocity (mi/h)	17	21	24	29	32	31	28